UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Examination in:	MEK4540/9540 – Composite Materials and Structures
Date of examination:	Monday 07-12-2009
Examination hours:	0900 – 1200
This examination paper consists of 3 pages	
Appendix:	Useful formulae (1 page)
Permitted aids:	Rottmann's formula compilation + approved calculator

Make sure that your copy of this examination paper is complete before answering.

Problem 1 (30%)

a) Explain the difference between *thermoset* and *thermoplastic* resins for use as matrix materials, and give examples of each type.

b) Describe briefly, with the aid of sketches, two different production methods that can be used for fibre composites, and indicate some of the advantages and disadvantages of these methods.

c) A uni-directional composite consists of fibres with Young's modulus E_f and a matrix with Young's modulus E_m. The fibre volume ratio is V_f. Derive expressions for the Young's moduli of the composite, E_L and E_T, in the directions parallel and perpendicular to the fibres, respectively. Explain the assumptions you make in the derivations and comment on their validity.

d) By considering the forces on an element of a fibre with circular cross-section in a short fibre composite under tensile loading parallel to the fibre axis, show that

$$\frac{d\sigma_f}{dz} = \frac{4\tau}{d}$$

where σ_f is the axial stress in the fibre, τ is the shear stress acting between the matrix and the fibre, d is the fibre diameter and z is the distance along the fibre measured from one end of the fibre.

Hence show that an estimate of the load transfer length l_t is given by

$$\frac{l_t}{d} = \frac{\left(E_f / E_c\right)\sigma_c}{2\tau_y}$$

where E_f is the fibre elastic modulus, σ_c is the stress applied to the composite in the direction parallel to the fibre, E_c is the elastic modulus of a long-fibre uni-directional composite with the same fibre volume fraction, and τ_y is the yield stress of the matrix in shear. State clearly the assumptions made in the derivation.

Problem 2 (30%)

PART A

a) A specially orthotropic material has elastic constants E_L, E_T, G_{LT}, ν_{LT} and ν_{TL}. By writing expressions for the strains induced under individual stress states, and superposing these contributions, show that Hooke's law for plane stress conditions can be written:

$$\begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{bmatrix} = [S] \begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{bmatrix} \quad \text{where the compliance matrix} \quad [S] = \begin{bmatrix} \dfrac{1}{E_L} & -\dfrac{\nu_{TL}}{E_T} & 0 \\ -\dfrac{\nu_{LT}}{E_L} & \dfrac{1}{E_T} & 0 \\ 0 & 0 & \dfrac{1}{G_{LT}} \end{bmatrix}$$

b) The stiffness matrix for a laminate consists of sub-matrices A, B and D which are defined in the attached sheet of formulae. What is the physical interpretation of each of these sub-matrices? Under what conditions is the sub-matrix B zero? What is the significance of the elements A_{16} , A_{26} and D_{16}, D_{26}, and under what conditions are they zero?

PART B

A laminate consists of plies with unidirectional fibres in a thermoset matrix. The material in each ply has the following elastic properties:

$$E_L = 26.7\,\text{GPa}, \quad E_T = 8.4\,\text{GPa}, \quad G_{LT} = 3.5\,\text{GPa}$$
$$\nu_{LT} = 0.29, \quad \nu_{TL} = 0.091$$

a) Determine the compliance matrix [S] and the stiffness matrix [Q] in the LT coordinate system for such a ply.

b) A laminate is laid up with the configuration [0/90/90/0] with respect to the x-axis in a global coordinate system x-y. Each ply has thickness 1.2 mm. Calculate the stiffness matrix $[\overline{Q}]$ for a 90° ply with respect to the global coordinate system, and the A-matrix for the laminate.

c) The laminate in part b) is exposed to an in-plane loading. The following strains are measured with strain gauges in the global x-y coordinate system:

$$\varepsilon_x = 1500 \times 10^{-6} \ ; \ \varepsilon_y = 500 \times 10^{-6} \ ; \ \gamma_{xy} = 800 \times 10^{-6}$$

Calculate the in-plane loading (N_x, N_y, N_{xy}) that has induced these strains.

d) How could you calculate the elastic moduli E_x and E_y in the global x- and y-directions and the shear modulus G_{xy} for such a laminate?

Problem 3 (40%)

PART A

Describe briefly, with the aid of sketches, the failure mechanisms that may occur when a sandwich beam is subjected to either transverse or axial loading.

PART B

a) The following four equations are commonly used as the basis for calculating the deformations of sandwich beams under lateral loading:

$$w = w_b + w_s \ ; \quad -D\frac{d^2 w_b}{dx^2} = M_x \ ; \quad T_x = \frac{dM_x}{dx} \ ; \quad T_x = S\frac{dw_s}{dx} - \gamma_0 \frac{t_c}{d}$$

The symbols w, M_x and T_x refer to the transverse displacement, the bending moment per unit width and the transverse shear force per unit width, at a distance x from one end of the beam. Symbols t_c and d refer to the core thickness and the distance between the mid-surfaces of the face sheets, respectively.

Explain the meaning of each of the other symbols w_b, w_s, D, S and γ_0. Explain also the significance of each equation. Under what conditions is $\gamma_0 = 0$?

b) Figure 1 shows a horizontal sandwich cantilever beam, AB, with length L, built in at A. A vertical, uniformly distributed load q per unit area is applied over the entire length of the beam. Both face sheets can be considered as thin and the core as weak (compliant).

With the aid of the equations in a) above, find an expression for the vertical displacement w as a function of x and show that the displacement δ at the end B of the beam is given by

$$\delta = \frac{qL^4}{8D}\left(1 + \frac{4D}{SL^2}\right)$$

c) Describe briefly how the analysis to find $w(x)$ would have to be modified if end B were simply supported instead of being free.

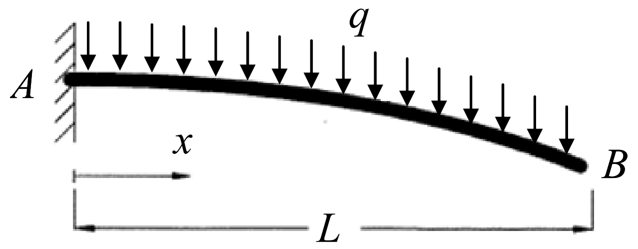
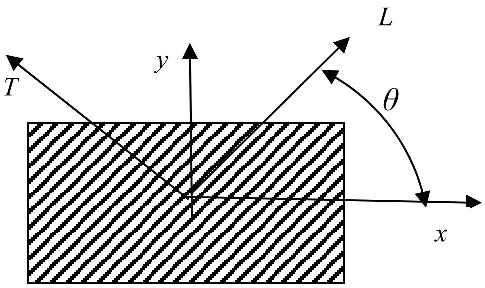


Figure 1: Sandwich cantilever beam with uniformly distributed load.

USEFUL FORMULAE

$$[T]=[T_1]=\begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2-s^2 \end{bmatrix}$$



$$[T_2]=\begin{bmatrix} c^2 & s^2 & sc \\ s^2 & c^2 & -sc \\ -2sc & 2sc & c^2-s^2 \end{bmatrix}$$

$c = \cos\theta$; $s = \sin\theta$

Ply properties using tensor strains	Ply properties using engineering strains
$\begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \frac{1}{2}\gamma_{LT} \end{bmatrix} = [S^t]\begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{bmatrix}$ where	$\begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{bmatrix} = [S]\begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{bmatrix}$ where
$[S^t]=\begin{bmatrix} \frac{1}{E_L} & -\frac{\nu_{TL}}{E_T} & 0 \\ -\frac{\nu_{LT}}{E_L} & \frac{1}{E_T} & 0 \\ 0 & 0 & \frac{1}{2G_{LT}} \end{bmatrix}$	$[S]=\begin{bmatrix} \frac{1}{E_L} & -\frac{\nu_{TL}}{E_T} & 0 \\ -\frac{\nu_{LT}}{E_L} & \frac{1}{E_T} & 0 \\ 0 & 0 & \frac{1}{G_{LT}} \end{bmatrix}$
$[Q^t]=[S^t]^{-1}$	$[Q]=[S]^{-1}$
$\begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{bmatrix} = [T]\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$	$\begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{bmatrix} = [T_1]\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$
$\begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \frac{1}{2}\gamma_{LT} \end{bmatrix} = [T]\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{1}{2}\gamma_{xy} \end{bmatrix}$	$\begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{bmatrix} = [T_2]\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$
$[\overline{Q}^t]=[T]^{-1}[Q^t][T]$	$[\overline{Q}]=[T_1]^{-1}[Q][T_2]$

$[\overline{Q}]$ can be obtained by dividing each term in the 3rd column of $[\overline{Q}^t]$ by 2.

$$A_{ij} = \sum_{k=1}^{n}\left(\overline{Q}_{ij}\right)_k\left(h_k - h_{k-1}\right) \;;\quad B_{ij} = \frac{1}{2}\sum_{k=1}^{n}\left(\overline{Q}_{ij}\right)_k\left(h_k^2 - h_{k-1}^2\right) \;;\quad D_{ij} = \frac{1}{3}\sum_{k=1}^{n}\left(\overline{Q}_{ij}\right)_k\left(h_k^3 - h_{k-1}^3\right)$$